

[10/21/24]

## RECURSION vs. ITERATION - when to use recursion

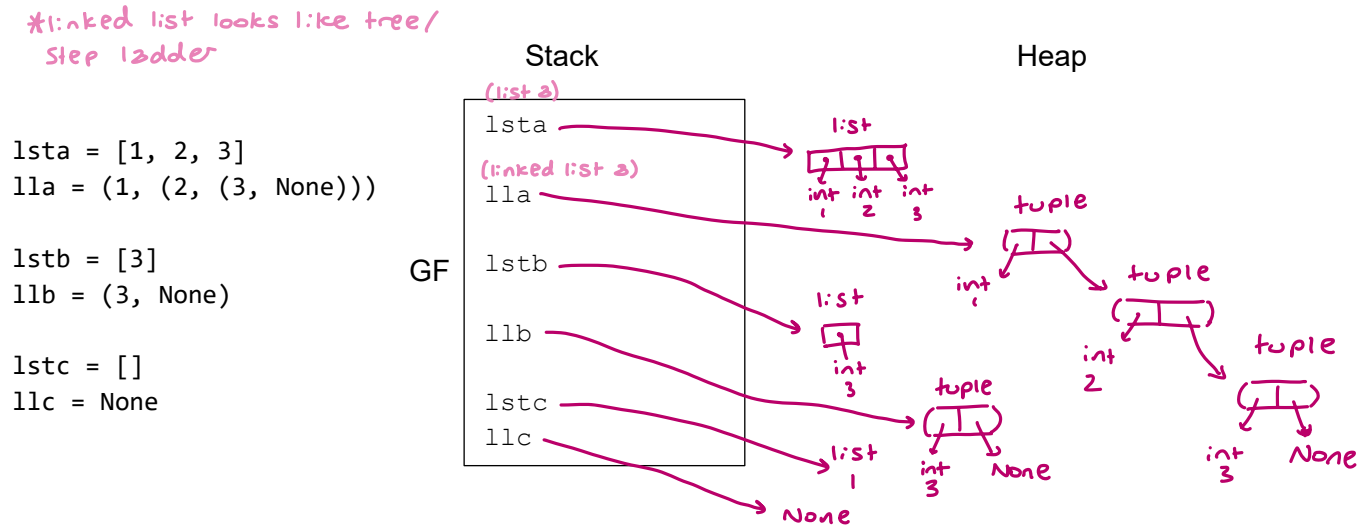
- natural recursion problem definition
- tree-like data
- list-like data

## LINKED LIST vs. LIST:

- linked list tuple of tuples is IMMUTABLE!
- regular list: have to copy before copying
- list[1:] takes longer than linked-list[1]
- linked list better for recursion
- linked list is hard to index into (list[2] vs. linked-list[1][1][0])
- linked list hard to find true len (len(list) = #, len(linked-list) = 2)

**Question 1:** For today's recitation we will define an empty linked list as `None`, and a non-empty linked list as a length two tuple of `(element, linked_list)`.

Complete the environment diagram to represent the execution of the code below.



**Question 2:** Fill in the body of the functions below:

*import doctest*

def first(ll):

"""

returns the first element of a non-empty linked list

>>> first( (5, (10, (15, None))) )

5

"""

*return ll[0]*

*doctests!!*

*good things to be aware of*

*(provides documentation AND testing)*

*→ there's a way to run all doctests or pick a certain one to run*

def rest(ll):

"""

returns the rest of a nonempty linked list  
(omitting the first element)

>>> rest( (5, (10, (15, None))) )

(10, (15, None))

"""

*return ll[1]*

Why would these helper functions be useful?

*• good for clarity*

*•*

**Question 3:** Implement the following functions recursively, and then iteratively:

```
def ll_len(ll):
```

```
    """
```

```
    get the length of a linked list
```

```
>>> ll_len( ('a', ('b', None)) )
```

```
2
```

```
    """
```

```
    # RECURSION:
```

```
    if ll is None: # if not ll
```

```
        return 0
```

```
    # ITERATIVELY:
```

```
    count = 0
```

```
    while ll is not None:
```

```
        ll = rest(ll)
```

```
        count += 1
```

```
    return count
```

# How DOES THIS NOT  
MUTATE ll??

points @  
different tuples  
but not changing  
b/c not assign  
etc

base  
case

recursion → return 1 + ll\_len(ll[1]) # or: return 1 + ll\_len(rest(ll))

helper function  
we wrote

RECURSION:

ll\_len( ('a', ('b', None)) ) → 2

↳ 1 + 1

ll\_len( ('b', None) )

↳ 1 + 0

ll\_len( None )

↳ 0



```
def ll_get(ll, i):
```

```
    """
```

```
    get the ith element of a linked list
```

```
>>> ll_get( ('a', ('b', None)), 1)
```

```
'b'
```

```
    """
```

```
    # RECURSIVE:
```

```
    if i >= len(ll) or i < -len(ll) # if index too big OR you've added the length to negative  
        raise IndexError('linked list index {i} out of range')
```

```
    if i < 0:
```

```
        i += len(ll)
```

```
    if i == 0:
```

```
        return first(ll)
```

```
    return ll_get(rest(ll), i-1)
```

BASE  
CASE

\* can use online to reference  
doctests & what is happening

```
original-i = i  
length = 11-len(ll)
```

```
if i >= len(ll) or i < -11-len(ll)  
    raise IndexError(f'linked list index {original-i} out of range')  
return helper(rest(ll), i-1)
```

```
def helper(ll, i):  
    if i == 0:  
        ... -  
        etc.
```

**HOISTING!** having recursive stuff in  
seperate function

[10/23/24]

### WHY LINKED LIST?

- minesweeper lab has lots of nested lists - linked lists good for nested structures
- inherently recursive structure - easier to split
- `list[1:]` makes new copy while `linked-list[1]` does NOT make copy
- linked list immutable
- LISP implements linked list



*\*order matters*  
`make_ll(n, *elements) → make_ll(1, 2, 3) → n=1, elements=(2,3)`

**Question 1:** Implement the following function recursively, and then iteratively.

```
def make_ll(*elements):
    """
```

*makes a tuple of the elements  
(unpacking) → make\_ll(1, 2, 3) → elements = (1, 2, 3)  
make\_ll() → elements = ()*

given an arbitrary number of elements as arguments,  
make a linked-list of (first, rest) pairs

```
>>> make_ll(1, 2, 3)
(1, (2, (3, None)))
"""
```

*#RECURSIVE*

```
if not elements: ← if elements is empty list
    return None
```

```
return (elements[0], make_ll(*elements[1:]))
```

*#ITERATIVE:*

```
ll = None
for elt in reversed(elements):
    ll = (elt, ll)
return ll
```

*doesn't create a copy while elements[::-1] does*

*elements (1, 2, 3)*

*make\_ll((2, 3))*

*(2, 3), —*

*w/o star, will add element tuple into linked list instead of individual el.s.*

**Question 2:** Implement the following function recursively, and then iteratively.

```
def ll_concat(ll1, ll2):
    """
```

return a new linked list that concatenates two linked lists

```
>>> ll_concat(make_ll(1), make_ll(2, 3))
(1, (2, (3, None)))
>>> ll_concat(None, make_ll(4, 5))
(4, (5, None))
"""
```

*#RECURSIVE:*

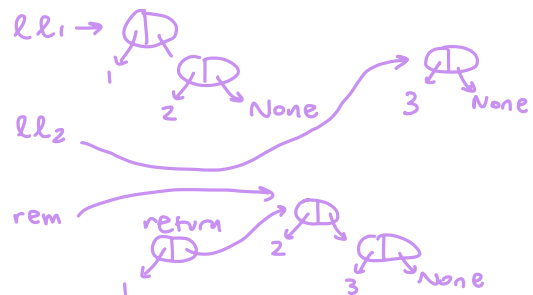
```
if ll1 is None:
    return ll2
if ll2 is None:
    return ll1
else:
    remainder = ll_concat(rest(ll1), ll2)
    return (first(ll1), remainder)
```

*like adding empty list to list → [7] + [4, 5]*

*#ITERATE - much messier!*

```
new_ll = ll2
for i in reversed(range(ll_len(ll1))):
    new_ll = (ll_get(ll1, i), new_ll)
return new_ll
```

*\*NOTE: doctests will FAIL if you have print statements!!*





**Question 3:** Implement the following function recursively, and then iteratively.

```
def ll_reverse(ll, so_far = None):
```

```
    """
```

```
    return a new reversed linked list
```

```
>>> ll_reverse(make_ll(1,2,3))
```

```
(3, (2, (1, None)))
```

```
    """
```

```
    #RECURSIVE
```

```
    if not ll:
```

```
        return None
```

```
    return ll_reverse(rest(ll), (first(ll), so_far))
```

```
    #ITERATIVELY
```

```
    new_ll = None
```

```
    while ll:
```

```
        new_ll = (elt, ll)
```

```
        ll = rest(ll)
```

```
    return new_ll
```

← can't do for loop  
(or else would just be a linked list then stop)

**Question 4:** Implement the following function recursively, and then iteratively.

```
def ll_elements(ll):
```

```
    """
```

```
    return a generator that yields each element in a linked list
```

```
>>> ll_gen = ll_elements(make_ll(1, 2, 3))
```

```
>>> next(ll_gen)
```

```
1
```

```
>>> list(ll_gen)
```

```
[2, 3]
```

```
    """
```

```
    while ll:
```

```
        yield first(ll)
```

```
        ll = rest(ll)
```

GENERATOR: play/pause button.

-yields then will keep playing

-NO element is returned until

next() is called

- generator is created then stays there

```
ll_gen = ll_elements(make_ll(1,2,3))
```

↳ generator object (doesn't show object)

```
print(next(ll_gen)) ← forces ll_gen to run until yield
```

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REFACTORING: improving code w/o changing its function

CODE SMELLS: some engineers talk ab. code having good/bad "smell" (syn. for style)

3 R'S OF REFACTORING:

→ READABILITY (documentation + style)

→ AVOID REPETITION (Don't repeat yourself, make helper functions, etc.)

→ CONSIDER RUNTIME (efficiency)

REFACTOR PROCESS:

- 1) understand
- 2) make a plan
- 3) implement plan
- 4) look back

**Question 1:** What strategy did you use when refactoring the 2d-version of minesweeper?

```
def dig_2d(game, row, col, check_victory = True)
    # game is over, do nothing
    if game["state"] == "defeat" or game["state"] == "victory":
        game["state"] = game["state"] # keep the state the same
        return 0
    if game["state"] != "ongoing":
        return 0
```

# checking if we dug a mine & lost

```
board_val = game["board"][row][col]
visible_val = game["visible"][row][col]
if game["board"][row][col] == ".":
    game["visible"][row][col] = True
    game["state"] = "defeat"
    return 1 # revealed
```

# game state check - don't need to do it before

```
num_revealed_mines = 0
num_revealed_squares = 0
for r in range(game["dimensions"][0]):
    for c in range(game["dimensions"][1]):
        if game["board"][r][c] == ".":
            if game["visible"][r][c] == True:
                num_revealed_mines += 1
            elif game["visible"][r][c] == False:
                num_revealed_squares += 1
if num_revealed_mines != 0:
    # if num_revealed_mines is not equal to zero, set the game state to
    # defeat and return 0
    game["state"] = "defeat"
    return 0
if num_revealed_squares == 0:
    game["state"] = "victory"
    return 0
```

```
visible_val
if game["visible"][row][col] != True:
    game["visible"][row][col] = True
    revealed = 1
else:
    return 0
```

```
board_val
if game["board"][row][col] == 0:
    nrows, ncolumns = game["dimensions"]
    if 0 <= row - 1 < ncolumns:
        if 0 <= col - 1 < ncolumns:
            if game["board"][row - 1][col - 1] != ".":
                if game["visible"][row - 1][col - 1] == False:
                    revealed += dig_2d(game, row - 1, col - 1)
            # ... some code that was copy / paste / modify omitted
        if 0 <= row + 1 < ncolumns:
            if 0 <= col + 1 < ncolumns:
                if game["board"][row + 1][col + 1] != ".":
                    if game["visible"][row + 1][col + 1] == False:
                        revealed += dig_2d(game, row + 1, col + 1)
```

# checking  
for 9 neighbors  
surrounding  
the board value

already making sure there are no  
mines around

outside of loop already setting visible board to True

```
for r in range(max(row-1, 0), min(nrows, row+2)):
    for c in range(max(col-1, 0), min(ncolumns, col+2)):
        revealed += dig_2d(game, r, c)
```

Combining oob  
condition w/  
getting neighbors

```
num_revealed_mines = 0 # set number of mines to 0
num_revealed_squares = 0
for r in range(game["dimensions"][0]):
    # for each r
    for c in range(game["dimensions"][1]):
        # for each c
        if game["board"][r][c] == ".":
            if game["visible"][r][c] == True:
                # if the game visible is True, and the board is '.',
                # add 1 to mines revealed
                num_revealed_mines += 1
            elif game["visible"][r][c] == False:
                num_revealed_squares += 1
        return revealed
```

```
if game["board"][r][c] != '.':
    and not game["visible"][r][c]:
        return revealed
```

Short  
Circuiting

?? bad\_squares = num\_revealed\_mines + num\_revealed\_squares

```
if bad_squares == 0:
    game["state"] = "ongoing"
    return revealed
else:
    game["state"] = "victory"
    return revealed
```

**EFFICIENCY:**

when making smthg. a set, it takes a long time (as long as # elements in set)

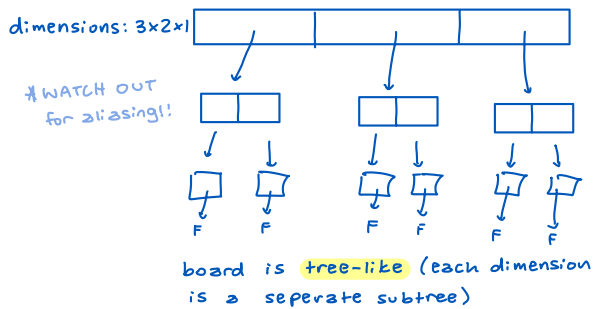
```
mines = set(tuple(mine) for mine in mines)
for r, c in mines:
```

create set outside the  
loop to reduce runtime

storing values vs. function calls



**Question 2:** What are instances of tree-like, graph-like, and list-like recursion in the mines lab?



#### GRAPH-LIKE:

looking for neighbors if the board value is 0

→ the 'visited set' is the visible board in this case  
(to prevent inf. recursion of going back/forth)

#### LIST-LIKE:

→ get/set value to peel off the 1st coordinate (dimension)  
first: coord[0], rest: coord[1:]

→ get neighbors

→ get all coordinates

**Question 3:** Below is a recursive all\_coords function that returns a list of tuple coordinates. Modify the code below to make this function into an efficient generator.

```
def all_coords(dimensions):
    """
    A function that generates all possible coordinates in a given board.
    """
    if len(dimensions) == 1:
        return [(x,) for x in range(dimensions[0])]

    first = all_coords(dimensions[:1])
    rest = all_coords(dimensions[1:])
    result = []
    for start in first:
        for end in rest:
            result.append(start + end)
    return result
```

## 10/30/24 - BACKTRACKING

### 1) UNDERSTAND the problem

- what am i trying to satisfy?
  - what are my choices?
- what are my constraints?
- draw decision tree

### 2) make the plan

- use 'recipe,' fill in blanks

### 3) implement the plan (code)

### 4) look back (optimize)

**Question 1:** You ordered food from SuperEats for you and your friends. SuperEats delivered a variety of entrees with varying quantities. Your friends have given you their unordered preferences for which entrees they like. As the host, you are trying to determine a way to assign the delivered food to your so they can all get one of their preferred dishes.

No solution example (there is not enough food for everyone):

SATISFY: getting the food

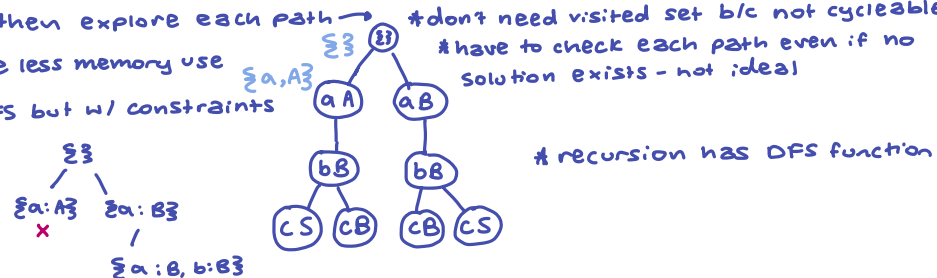
```
people = {'alex': ['Acai', 'Burger'],
          'bob': ['Burger'],
          'cam': ['Burger', 'Salad']}
food = {'Burger': 2, 'Salad': 0, 'Acai': 0}
```

Solution example (alex and bob can get burgers and cam can get a salad):

```
people = {'alex': ['Acai', 'Burger'],
          'bob': ['Burger'],
          'cam': ['Burger', 'Salad']}
food = {'Burger': 2, 'Salad': 1, 'Acai': 0}
```

Discuss with someone around you how you would approach solving this problem using different graph search methods:

- brute force search → generate all possibilities then check each
- BFS → start from a person then explore each path
- DFS → pop(-1) instead; will be less memory use
- Backtracking → similar to DFS but w/ constraints



### BRUTE FORCE (exhaustive)

solution:

- explore all food items/pp!

### BFS:

- remove nodes from opposite sides of agenda
- explores shortest paths 1st

### DFS:

- explore current then backtrack

### BACKTRACK

- similar to DFS but backtrack as soon as we realize it's not worth continuing down some branch
- good for problems w/ constraints

**Question 2:** Fill in the body of the feed function below.

```
def feed(people, foods):
    """
```

Given people who are hungry and the available food supplies, find a mapping from people to available foods they prefer if one exists.

Parameters:

people: a dictionary mapping a name to a list of their preferred foods  
 food: a dictionary mapping available foods to their quantities

Returns:

Dictionary mapping person to assigned food if there is enough food to match everyone's preferences. None otherwise.

```
>>> people = {'alex': ['oreo', 'chocolate'], 'bobbie': ['vanilla']}
>>> feed(people, {'oreo': 1, 'vanilla': 1}) == {'alex': 'oreo', 'bobbie': 'vanilla'}
True
>>> feed(people, {'oreo': 1, 'ketchup': 1}) == None
True
"""
# if nothing left to satisfy -> SUCCESS!!
if not people:
    return {}

# choose one thing to satisfy
person = min(people, key=lambda p: len(people[p])) # take the most constrained person (min len)
person = next(iter(people)) # same as list(people.keys())[0]

# look through choices to satisfy
for food in people[person]: # food has to be in person preferences
    # non-zero food
    if foods.get(food, 0) > 0: # prevents KeyError (default 0)
        people.pop(person)
        new_people = {p: f for p, f in people.items()}
        new_foods = {f: i for f, i in foods.items()}
        new_foods[food] -= 1 # -1 food available
        result = feed(new_people, new_foods)
        if result is not None: # success
            result[person] = food
            return result

# if no valid choices - FAILURE
return None
```



## SAT Solver Overview:

→ satisfying assignment w/ backtracking recipe

→ optimizing

- prune tree
- avoid cycles
- avoid extra copying

 $f = (a \text{ or } b \text{ or } c) \text{ and } (b \text{ or not } a) \text{ and } (\text{not } b) \leftarrow \text{CNF}$ 

Satisfying-assignment (SA)

RULES: \* trying to satisfy the clause:

- no clauses left → solved!
- empty clause = unsolvable
- variable can only have one value

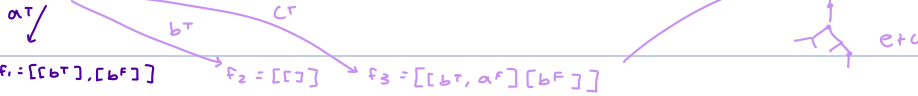
\* computers NOT fast enough

to keep up w/  $2^n$ 

(try to minimize tree branches)

SA(F)

var, val = 'a', True



SA(F1)

bT

f2 = [[]] X backtrack

SA(F3)

bT

[[]] X

aF

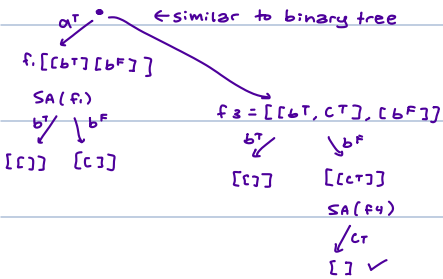
[[bT]] etc.

nondeterministic  
polynomial timepolynomial  
time

NP = P?

 $n^k \lll 2^n$ much more  
efficient\* INEFFICIENT b/c unit clause  $B^F$  MUST work!

· loop through unit clauses or the T/F of shortest clauses



## YIELD VS. YIELD FROM:

· yield from is basically a for loop + yield

def my\_gen():

for e1 in [1, 2, 3]:

yield e1

} gets 1, 2, 3

yield from [1, 2, 3] } also gets 1, 2, 3

· yield returns a single val from generator function

· yield from yields all values from iterator/gen.

**Question 1:** Applying Polya's problem solving method to satisfying\_assignment:

`format: ((P,F), True)`

1] Understand the problem

- What are the rules / constraints of satisfying\_assignment?
- What are we trying to satisfy?
- What kind of decision tree do we end up with?

*↑ instead of combine  
(ppi name kids we'd things)*

*similar to boolify:*

*· food mapped to quantity similar to room to capacity*

*· name & preferred food*

*Rules:*

*1: each person only needs 1 food*

*2: can only give out x number of food*

*3: ppi need to eat exactly 1 food*

2] Make a plan

- How do we apply our general recipe for backtracking to satisfying\_assignment?
- Will we need any helper functions to be useful?

*→ preferred\_foods() (rule 1)*

*→ 1 function for rule 2 person\_one\_food(), rule 3 at-most()*

*with format ((person, food), bool), how to  
take result of satisfying\_assignment and  
output final result?*

3] Implement the plan

*if result is None:*

*return None*

*else:*

*assign = {}*

*for (people, food), value in result.items():*

*if value:*

*assign[people] = food*

*return assign*

4] Look back

- How can we optimize this search?

**Question 2:** What is the result of calling satisfying\_assignment on the formula below? How does using the unit clause optimization make solving this formula faster?

```
formula = [
    [('a', True), ('b', True), ('c', True)],
    [('b', True), ('a', False)],
    [('b', False)],
]
result = satisfying_assignment(formula)
```

**Question 3:** How can we solve our potluck from last Wednesday using `satisfying_assignment`?

**Question 4:** implement `all_combinations` as a generator function

```
def all_combinations(elements, N):
    """
    Given a list of hashable elements (with no duplicates) and N
    (the size of each combination), make a generator that outputs
    length-N tuples of all combinations of the elements

    >>> sorted(all_combinations([1, 2, 3, 4], 0)) == [()]
    True
    >>> sorted(all_combinations([1, 2, 3, 4], 1)) == [(1,), (2,), (3,), (4,)]
    True
    >>> y = all_combinations([1, 2, 3, 4], 2)
    >>> sorted(y) == [(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)]
    True
    >>> z = all_combinations([1, 2, 3, 4], 3)
    >>> sorted(z) == [(1, 2, 3), (1, 2, 4), (1, 3, 4), (2, 3, 4)]
    True
    >>> sorted(all_combinations([1, 2, 3, 4], 4)) == [(1, 2, 3, 4)]
    True
    >>> sorted(all_combinations([1, 2, 3, 4], 5)) == []
    True
    """
```

with generators:  
think about what you  
want to append!!

\*if generators are scary,  
return list first then  
do a generator that  
yields from

GENERATOR:

```
if N == 0:
    yield [()]
elif not elements:
    yield []
else:
    first = elements[0]
    rest = elements[1:]
    yield from all_combinations(rest, N)
    for combo in all_combinations(rest, N-1):
        yield (first,) + combo
```

NOT GENERATOR:

```
if N == 0:
    return [()]
elif not elements:
    return []
else:
    first = elements[0]
    rest = elements[1:]
    all_combos = all_combinations(rest, N)
    for combo in all_combinations(rest, N-1):
        all_combos.append((first,) + combo)
    return all_combos
```

WHEN TO USE self. — ?

→ when you want to keep a variable

when print(obj) in a class, calls — str —

— repr — also

abstraction, planet simulation

**Question 1:** Today we're going to build a planet physics simulation (see video.) Discuss with a neighbor: what classes would be a good idea to implement? What attributes and methods should each class have?

$$\vec{v}_i[t+1] \approx \vec{v}_i[t] + \vec{a}_i[t] dt$$

$$\vec{x}_i[t+1] \approx \vec{x}_i[t] + \vec{v}_i[t] dt$$

$$\vec{a}_i = \frac{\sum_j \vec{F}_{ij}}{m_i} \rightarrow \vec{F}_{ij} = \frac{G \cdot m_i \cdot m_j}{\|r\|^2} \hat{r}$$

CLASSES:

Planet

Simulation

ATTRIBUTES:

mass <sup>2D space</sup>  
 ↓  
 position (vector)  
 velocity

list of planets

nd coord list

METHOD:

• forces  
 - parameters: planet, list of other planets applying  $\vec{F}$

apply timestep dt

+, -, /, \*

→ vector  
 will implement these today

**Question 2:** Fill in the missing code below for the following Vector class methods:

```

class Vector:
    # first arg. always the instance of self
    def __init__(self, coords):
        # each Vector object has a nd tuple of coords
        self.coords = tuple(coords)

    def __repr__(self):
        # repr(Vector([0, -4])) -> 'Vector((0, -4))'
        return f'__self__.__class__.__name__({self.coords})'

    def add(self, other):
        # can use + instead of .add b/c DUNDER!!
        # Vector([1, 2]).add(Vector([1, 0])) -> Vector((2, 2))
        return Vector([i+j for i,j in zip(self.coords, other.coords)])

    def sub(self, other):
        # Vector([1, 2]).sub(Vector([1, 0])) -> Vector((0, 2))
        return Vector([i-j for i,j in zip(self.coords, other.coords)])

    def scale(self, num, other):
        # Vector([1, 2]).scale(5) -> Vector((5, 10))
        return Vector([i*num for i,j in self.coords])

    def div(self, num, other):
        # Vector((4, 2)).div(2) -> Vector((2.0, 1.0))
        # .truediv
        return Vector([i/num for i,j in self.coords])
        OR
        return self.scale(1/num)

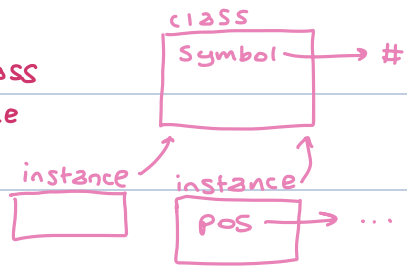
    def magnitude(self):
        # Vector((3, 4)).magnitude() -> 5.0
        # abs(vec....)
        return sum([i**2 for i in self.coords])**0.5

    def normalize(self):
        # creates a unit vector in the same direction
        # Vector([3, 4]).normalize() -> Vector([.6, .8])
        return self.div(self.magnitude())

```

always ask:  
what is input/output type?

CLASS ATTRIBUTE → stored inside class  
(each instance of class will share the  
class attribute)



**Question 1:** Eat That Sock is a 2D game where a human player competes with a bot to see who can score the most points in twenty seconds. Players score points by collecting socks. Socks can be different colors, with each color corresponding to a different point value. Socks randomly appear in the game, and also randomly disappear if the players don't collect them fast enough. Walls keep the players and socks in bounds.

Discuss with a neighbor: What are the different objects that we would need to represent this game? What attributes and methods would each object need?

### For example Game:

- Stores all other objects
- Keeps track of time remaining
- Each time step:
  - Moves players
  - Makes socks appear / disappear
  - Renders updated game board
- Decides winner at the end

\*think of classes & subclasses like a tree - parent = root node & child is a lower node.

### Sock:

- position (row, col)
- color: red, yellow, green
- symbol: s
- points: red 1, green 2, yellow 3
- TTL (time until disappear)

### Player:

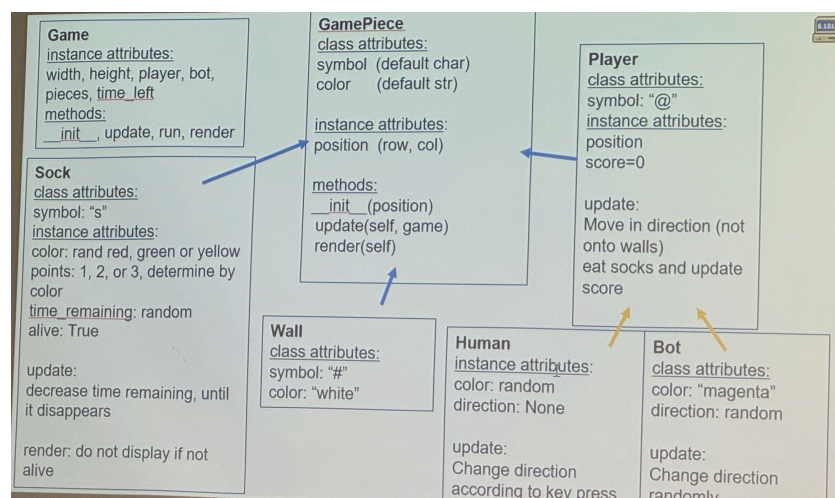
- position
- color (random)
- symbol: @
- render
- score
- change dir. w/ key
- move according to current dir.
- eats socks

### Bot:

- position
- color (magenta)
- symbol: @
- render
- score
- move according to dir.
- randomly change dir.
- eats socks

### Wall:

- position
- color (white)
- symbol: #
- render
- doesn't move
- can't intersect w/ other objects



GAMEPIECE  
class instances:  
color, symbol, alive





**Question 4:** Look at the blank Bot class shown below. Fill in the missing body. If implementing a method would be unnecessary, cross it out instead.

```
class Bot(Player):
    """
    A basic bot that randomly moves around the screen.
    """
    color = 'magenta'

def __init__(self, position):

def update(self, game):
    """
    Given the recent keys pressed and the current game, update the object.
    """
    directions = [Game.UP, Game.DOWN, Game.LEFT, Game.RIGHT]
    self.direction = random.choice(directions)
    Player.update(self, game)

def render(self):
    """
    Takes the state of the object at the end of a timestep and displays
    it to the screen.
    """
```

## R17 Participation Credit

Kerberos : \_\_\_\_\_@mit.edu

*Hand this sheet in at the end of recitation to get participation credit for today.*

**Question 2:** Look at the blank Wall class shown below. Fill in the missing body. If implementing a method would be unnecessary, cross it out instead.

```
class Wall(GamePiece):
    """
```

Static game piece represented by a white "#" symbol, which prevents pieces from going out of bounds.

```
    """
```

```
    symbol = "#"
```

```
    color = "white"
```

```
def __init__(self, position):
```

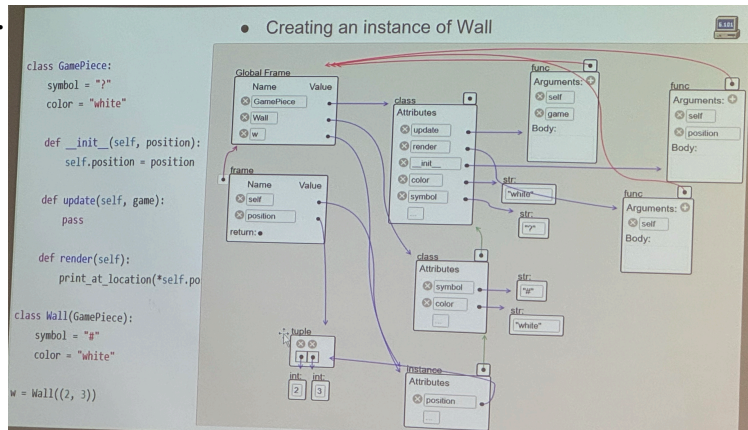
```
def update(self, game):
```

```
    """
    Given the current state of the game, update the object.
```

```
def render(self):
```

```
    """
    Takes the state of the object at the end of a timestep and displays
    it to the screen.
```

in render from class GamePiece:  
 print\_at\_location(\*self.position, self.symbol, self.color)



don't need to implement  
 any of these functions!!  
 all inherited from GamePiece

**Question 3:** Look at the blank Sock class shown below. Fill in the missing body. If implementing a method would be unnecessary, cross it out instead.

```
class Sock(GamePiece):
    """
    Static game piece represented by "s" that is visible for ttl timesteps
    before disappearing. Socks come in different colors, and each color is
    worth a different number of points. Players earn points if they intersect a
    sock before it disappears.

    Created by the game at a random position, initialized with a random color,
    disappears randomly.
    """
    symbol = 's'
    color_options = {'red': 1, 'green': 2, 'blue': 3}
    def __init__(self, position):
        self.color = random.choice(list(self.color_options.keys()))
        # super().__init__(position) works here, but NOT encouraged
        GamePiece.__init__(self, position)
        self.ttl = random.randint(10, 20)

    def update(self, game):
        """
        Given the current state of the game, update the object.
        """
        self.ttl -= 1
        if self.ttl <= 0:
            self.alive = False

def render(self): ← same code no matter what object
    """
    Takes the state of the object at the end of a timestep and displays
    it to the screen.
    """
```

GENERAL PURPOSE TOOLBOX: classes, functions, data structures

"NICHE" Tools: backtracking

WHAT IS TYPE-CHECKING:

Ex: every class has a precedence attribute (general class attribute)

- evaluate `_str_` w/ precedence instead of type-checking

- don't want to have to have many classes & many if statements

Ex: `Var('x') + 5` → `_add_`

`5 + Var('x')` → `_radd_`

IN EACH BINOP SUBCLASS... (add, sub, deriv, etc)

· simplify method

· eval method

MAKE-EXP: symbolic algebra vs. LISP

Symbolic algebra:

`(x + 5)`

`(x + 5 + y)`

LISP:

`(+ x 5)`

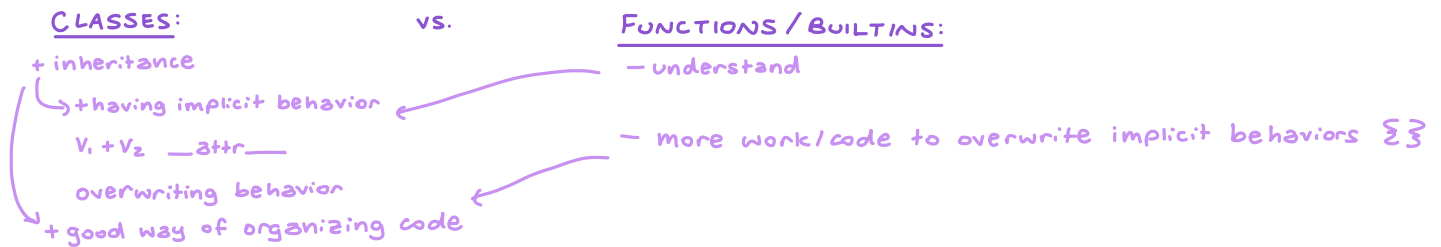
`(+ x 5 y)`

comments `(+ x 5); ignore rest of line`

TOKENIZE:

```
def tokenize(text):
    """
    tokens = []
    number_in_progress = ''
    for char in text:
        if '0' <= char <= '9': ← ascii (compares if char is
                                within these str ranges)
            number_in_progress += char
            continue ← keep looking for number in next ind
        if number_in_progress:
            tokens.append(number_in_progress)
            number_in_progress = ''
        if 'a' <= char <= 'z': # single chars
            tokens.append(char)
        elif char in '()+-*/': # separator
            tokens.append(char)
        elif char == ' ':
            continue ← ignore spaces
        else:
            raise Exception(f'unexpected character {char}')
```

**Question 1:** Discuss with a neighbor: What are the pros and cons of using classes (as opposed to implementing the same kind of behavior using builtin datatypes and functions)? What are some real-world applications of classes?



**Question 2:** Discuss with a neighbor: What are some real-world applications of inheritance?

- many shared behaviors / attributes b/t things (sound, pieces, animals, etc)
- Ex: instead of implementing Sounds as dicts... class Sound, MonoSound, StereoSound

Today we're going to think about how to implement a variation of make\_expression that has:

- nonnegative integers, like 3 or 10
- single-letter lowercase variables, like x or y or z
- fully-parenthesized binary operators, like  $(3*x)$  for +, -, /, and \*
- no extra parens or missing parens
- ignores spaces

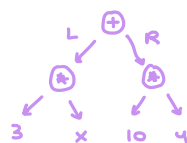
**Question 4:**

make\_expression(" ((3 \* x)+(10\*y) ) ") => Add ( Mul (Num(3), Var('x')), Mul (Num(10), Var('y')) )  
 → tokenized (split into chars in list)  
 → parsed



\* TREE-LIKE RECURSION!

- left, right branches off
- Var, Num are leaf nodes



**Question 5:** Discuss with a neighbor: How would we go about tokenize?

→ looking for separator or num  
→ example code above

**Question 6:** Discuss with a neighbor: How would we go about parse? What recursive pattern are we using (list-like, tree-like, graph-like)?

```
def parse(tokens): # tokens is a list of strings
```

```
    def parse_exp(index):
        # base cases = leaves of tree
        # if number
        if '0' <= tokens[index][0] <= '9':
            return Num(int(tokens[index])), index + 1
        # if b/t 2-3 (var)
        if 'a' <= tokens[index][0] <= 'z':
            return Var(int(tokens[index])), index + 1
        # left & right sides
        left_exp, index_2 = parse_exp(index + 1)
        operator = tokens[index_2] # + - * /
        right_exp, index_3 = parse_exp(index_2 + 1)

        # Find operators & assign
        op_lookup = {'+': Add, '-': Sub, '/': Div, '*': Mul}
        operator_class = op_lookup[operator]
        return operator_class(left_exp, right_exp), index_3 + 1

    return parse_exp(0)
```

Bonus exercises:

- Modify `make_expression` to allow for repeated operations in the same parens, i.e.,  
`make_expression('(3 + 4 + x + (y - 5 - 2 - z) + (3 * v))')`
- Generators: Modify `tokenize` to output a generator and modify `parse` to take in a generator.
- Modify `make_expression` to detect and raise a custom error when the expression is malformed, i.e.,  
`make_expression('(3 + 4')`

11/20/24:

### WHY USE INTERPRETERS (LISP):

- help understand languages you already know
- idea: interpreter is just another program

INTERPRETER → program converting high-level language to machine code & then executes it on the go

-ex: Scheme

COMPILER → "translator" - program converting a program in one language to another

### PYTHON VS. SCHEME (interpreter)

# python		; scheme
def mag(x,y):		(define (mag x y)
return sqrt(x*x + y*y)		(sqrt (+ (* x x)
magnitude(3,4)		(* y y) ) )
		)
		(magnitude 3 4)

NEXT LAB → conditional statements

- recursion
- can use for any coding language

~ LAB ~

PROGRAM → Tokenize → Parse → Evaluate → Output

Ex: '(-(+ 3 2) 5); test comment'

tokenize: ['(', '-', '(', '+', '3', '2', ')', '5', ')']

parse: ['-', ['+', 3, 2], 5]

evaluate: 0

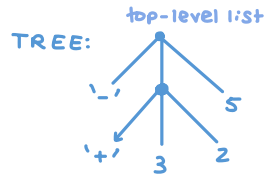
**Question 1:** Discuss with someone near you: what are the similarities and differences between tokenizing and parsing in the Symbolic Algebra lab and the LISP lab?

LISP:

- deals w/ comments ;
- can define a long, multi-letter variable w/ assignment
- evaluate different lines
  - parse/tokenize entire file

TREE-LIKE LISP

- leaf nodes (base exp): numbers / variables
- S-expressions: Ex: '(-(+3 2) 5); test comment'



EVALUATE:

[sum, 3, 2] → 5

[sub, 5, 5] → 0

EVALUATE:

- check type of the char & go from there
- str, num, list

[sum, 3, 2] → 5

[sub, 5, 5] → 0 ✓

**Question 2:** What will the code below output? Draw an environment diagram to represent the program execution.

```

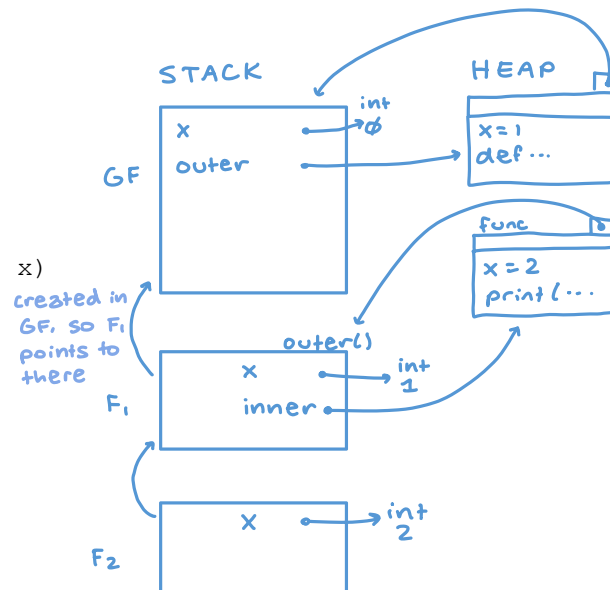
1  x = 0
2  def outer():
3      x = 1
4      def inner():
5          x = 2
6          print("inner:", x)
7
8      inner()
9      print("outer:", x)
10
11 outer()
12 print("global:", x)

```

```

inner: 2
outer: 1
global: 0

```



b) what if x=2 is commented out?

```

inner: 1
outer: 1
global: 0

```

← look into parent frame

c) if inner() function is moved out?  
(& x=2 still commented out?)

```

inner: 0
outer: 1
global: 0

```



Relatedly, what properties and methods would a Python class representing a frame object need? What about a function object?

FRAME CLASS

attribute:

- parent
- variables/values

methods:

- lookup (input: var return: value)

FUNCTION CLASS

attribute:

- enclosing frame
- parameters
- body

methods:

- call (input: arguments) ← python has custom \_\_call\_\_ method

\* can check if something is callable(...)  
(if has the call method)

1



**Question 3:** Rewrite each of the Python expressions below in Scheme.

# example 1

`(5 + 4) / (7 - 3 - 2 - 1) / 2`

`(/ (/ (+ 5 4) (- 7 3 2 1)) 2)`

★ # example 2

`(lambda x: x*x)(4)` ← function call surrounds

`((lambda (x) (* x x)) 4)` ←

★ # example 3

`def area(r):`

`return 3.14 * r ** 2`

`x = area(5)`

`y = x`

`(define area (lambda (r) (* 3.14 r r)))`

`(define x (area(5)))`

`(define y x)` ← var name is str, exp.

# example 4                      has ( ) around it

`def four():`

`return 2 + 2`

`four()`

`(define four () (+ 2 2))`

??  
`four`

11/25/24

LISP part 1 - diff. b/t function call & special form

WHY SCHEME?

- good practice to see similar/diff. b/t languages
- no looping in scheme! recursion only.
- useful for understanding test cases

**Question 1:** For each of the four statements written in Python below:

- What is the equivalent expression written in Scheme?
- What will the output of interpreting that expression be?
- How many times will evaluate be called in the course of interpreting that expression?

Note, example A has been completed for you.

; Example A:

; x = 4 ; provided Python code

(define x 4) ; Scheme equivalent

; output: 4

; # calls to evaluate: 2 - why? → evaluates x & evaluates 4

(lambda (x) (\* x x))

→ calls eval. once & stores the body

(+ 3 4 x)

→ eval. 5 times (looks @ each element)

; Example B:

; y = x - 1

; output: (define y (- x 1))

; # calls to evaluate: 3 (eval whole thing, then y & (- x 1))

; Example C:

; square = lambda s: s \* s

; output: (define s (lambda (s) (\* s s)))

; # calls to evaluate: 2 (for define & lambda)

(define (square x) (\* x x))

calls: 1 (just sets square to body)

; Example D:

; z = square(x) + square(y)

; output: (define z (+ (square x) (square y)))

; # calls to evaluate: 17 (square x creates new frame & calls eval. many times for the body, etc.)

(evaluator does a lot under the hood)

**Question 2:** The following Scheme code comes from test\_inputs/21.scm. Convert this scheme program into an equivalent Python program.

(define (call x) (x))	function object	
(call (lambda () 2))	2 ← creating func. & immediately calling	spam = lambda funct.
(define (spam) (call (lambda () 2)))	function object	spam()
(call spam)	2	call (spam) = 2
(call call)	Scheme Evaluation Error (bad args)	
(call)	Scheme Evaluation Error (missing arg)	

**Question 3:** Syntax errors occur when code breaks the rules the rules define the combinations of symbols that are considered to be correctly structured expressions in a programming language. Syntax errors are generally caught during parsing and prevent any lines of code from being interpreted / evaluated.

Discuss with a neighbor: what are common kinds of syntax errors in Python?

- indentation
- missing a colon

Thinking back to our Symbolic Algebra recitation last Monday, what are some examples of things that would cause syntax errors?

- missing round brackets
- can count parentheses, but don't have to.

As a reminder, our variation of `make_expression` last Monday handled expressions with:

- nonnegative integers, like 3 or 10
- single-letter lowercase variables, like x or y or z
- fully-parenthesized binary operators, like (3\*x) for +, -, /, and \*
- no extra parens or missing parens
- ignores spaces

# **SCHEME** **expr**

**atomic  
expression**

**var (str)**

**# (int, float)**

**S-expression**

**base  
exp**

**(func args...)**

**evaluate everything**

**built-ins**

**+ - / \* < >**

**<= >= not #t**

**#f equal?**

**special  
form**

**(define var expr)**

**(lambda (param.) expr)**

**if**

**and**

**or**

**Question 1:** Rewrite the `abs` function using a lambda expression in Python.

```
def abs(x):
    if x < 0:
        return -x
    else:
        return x
```

```
abs = lambda x: -x if x < 0 else x
```

**Question 1a:** Now write the `abs` function in Scheme.

```
(define abs (lambda (x)
  (if (< x 0)
      (* -1 x)
      x)))
```



**Question 2:** Why do `if`, `and`, and `or` need to be special forms in Scheme?

- important that both conditionals (T/F) don't get evaluated
  - `if`: only one of two options should happen
- `and` & `or` → short circuit (stops early)
  - `and`: stops as soon as F
  - `or`: stops as soon as T

**Question 3:** Write the `sign` function below in Scheme.

```
def sign(x):
    if x < 0:
        return -1
    elif x > 0:
        return 1
    else:
        return 0
```

```
(define sign x
  (if (< 0 x) -1
      (if (> 0 x) 1
          0)))
```

**Question 4:** Write `sum_squares` below in Python without using loops. This function should return the same input as the original version for all integers  $n \leq$  the recursion limit.

```
def sum_squares(n):
    total = 0
    for i in range(n+1):
        total += i*i
    return total
```

```
def sum_squares(n): # no loops!

    if n == 0:
        return 0
    return n*n + sum_squares(n-1)
```

**Question 4a:** Write `sum_squares` below in Scheme:

```
(define sum-squares n)
  (if (equal? n 0)
      0
      (+ (sum-squares (- n 1)) (* n n) )
  )

* begin statement
```



**Question 3a:** Write the `sqrt` function below in Python without using loops. This function should return the same input as the original version for all integers  $n \leq$  the recursion limit.

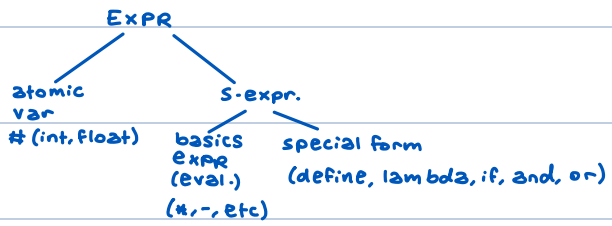
```
def sqrt(x, epsilon):
    guess = x / 2
    while abs(guess ** 2 - x) > epsilon:
        guess = (guess + x/guess) / 2
    return guess
```

```
def sqrt(x, epsilon):
    # your code below

    guess = x/2
    if abs(guess**2 - x) > epsilon:
        guess = (guess + x/guess)/2
        return sqrt(guess, epsilon)
    return guess
```

**Question 3b:** Write the `sqrt` function in Scheme.

12/4/24



#### PYTHON → SCHEME

- remove loops
- replace vars
- if: `__ if __ else __`



Today we'll be working with lists in Scheme, which will give us some practice with recursion and linked lists.

The table below shows the built-in list behavior that you will implement during Lisp part 2 and how it relates to Python list and linked tuple behavior that we have seen in recitation previously:

Scheme list	input	return type	Python list	linked tuple
(list? x)	any	boolean	isinstance(l, list)	
(list ...)	sequence<any>	linklist	[...]	make_ll(...)
(cons x y)	any	cons cell	[x, y]	(x, y)
(car x)	cons cell	any	x[0]	first(x)
(cdr x)	cons cell	any	x[1:], or x[1]	rest(x)
(append ...)	sequence<linklist>	linklist	sum(..., [])	
(length x)	linklist	int	len(x)	ll_len(x)
(list-ref x i)	linklist num	any	x[i]	ll_get(x, i)

**Question 1:** Write the `sum_list` function below in Scheme.

```
def sum_list(x): # x is a flat list of numbers
    out = 0
    for i in x:
        out += i
    return out
```

RECURSIVE:

return (0 if not x else x[0] + sum\_list(x[1:]))

```
(define (sum-list x)
  (if not x
      0
      (+ (car x) (sum-list (cdr x)))))
```

**Question 2:** Write the `sum_nested` function below in Scheme.

★ def sum\_nested(x): # x is a nested list of numbers

```
    out = 0
    for elt in x:
        out += sum_nested(elt) if isinstance(elt, list) else elt
    return out
```

```
(def (sum-nested x)
  (if (equal? (length x)
              0
              (+ (car x) (sum-nested (cdr x))))))
```

**Question 3:** Write the `subtract_elts` function below in Scheme.

```
def subtract_elts(x1, x2):
    # x1 and x2 are flat lists of numbers that have the same length
    result = []
    for i in range(len(x1)):
        result.append(x1[i]-x2[i])
    return result
```

SCHEME:

```
(define (subtract-elts x1 x2)
  (define (loop i)
    (if (equal? i (length x1))
        ()
        (list
         (- (list-ref x1 i)
            (list-ref x2 i))
         (loop (+ i 1))
        )
    )
  )
```

RECURSION:

```
def subtract_elts(x1, x2):
    def loop(i):
        if i == len(x1):
            return []
        else:
            return [x1[i] - x2[i]] + loop(i+1)
```

**Question 4:** Write the `find_max` function below in Scheme.

```
def find_max(x):
    # x is initially a non-empty nested list of numbers
    if isinstance(x, list):
        best = find_max(x[0])
        for elt in x:
            best = max(best, find_max(elt))
        return best
    return x
```

12/9/24 - last rec!

#### WE LEARNED:

- data structures (dict, sets, lists, iterables, generators)
- functional programming
- graph search
- recursion
- classes / inheritance
- languages / interpreters

#### COMMON EFFICIENCY BUGS:

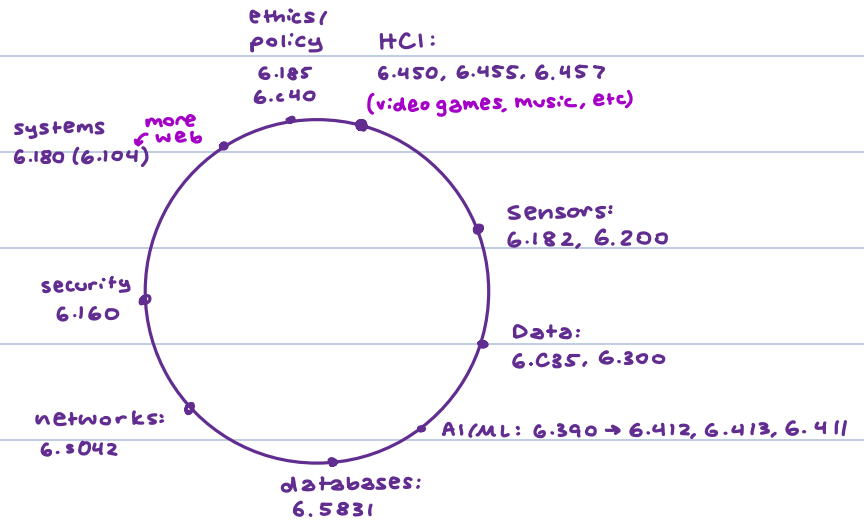
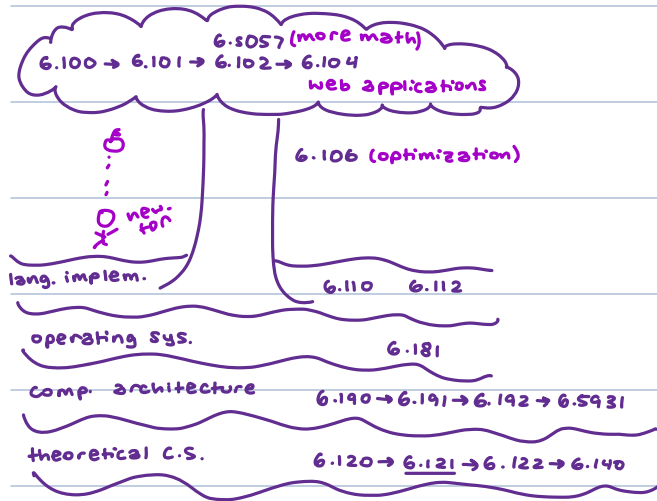
- superfluous computation
- suboptimal data structure design
- suboptimal algorithm design (SAD)

$x = 1:\text{st}(x)$

$\text{set } x = \text{set}(x)$   
conversions b/w  
types are LINEAR

#### LOOKING FORWARD:

- use imports
- python standard lib & external packages

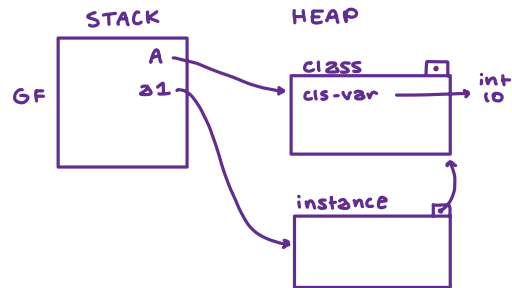


**Question 1: Classes + Environment Diagrams**

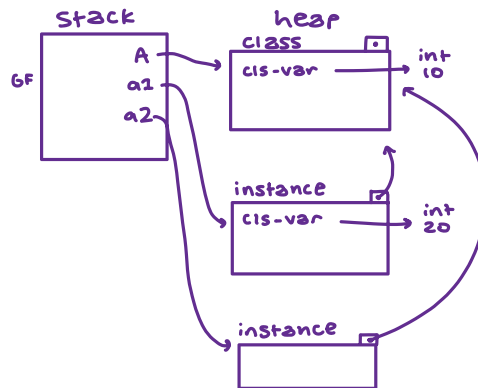
For each example program below, write what the output will be and draw the associated environment diagram. If the running the program would result in an error, write error instead and describe the problem.

**# example A**

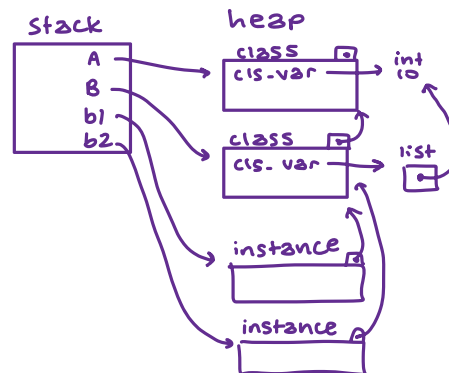
```
1 class A:
2     cls_var = 10
3     a1 = A()
4     print(a1.cls_var) # => 10
```

**# example B**

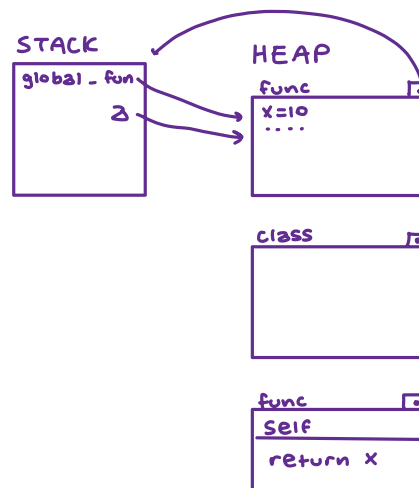
```
1 class A:
2     cls_var = 10
3     a1 = A()
4     a2 = A()
5     a1.cls_var = 20
6     print(a1.cls_var) # => 20
7     print(a2.cls_var) # => 10
```

**# example C**

```
1 class A:
2     cls_var = 10
3     class B(A):
4         cls_var = []
5
6     b1 = B()
7     b2 = B()
8     b1.cls_var.append(A.cls_var)
9     print(b1.cls_var) # => [10]
10
11    print(b2.cls_var) # => [10]
```

**# example D**

```
1 def global_fun():
2     x = 10
3     class A:
4         def a_fun(self):
5             return x
6     x = 20
7     return A()
8
9 a = global_fun()
10 print(a.a_fun()) # => _____
```



**Question 2: Recursion + Iteration + Generators**

For each example implementation of the map function below, what would the output of the following program be? If this would result in an error, write error instead.

```
def add_to_all(map_func, k, inp_list):
    return map_func(lambda n: n + k, inp_list)

print(add_to_all(map, 3, [1, 2, 3, 4, 5]))
print(list(add_to_all(map, 3, [1, 2, 3, 4, 5])))
```

```
# example A
def map(f, inp):
    return list(f(x) for x in inp)
```

```
# example B
def map(f, inp):
    yield f(inp[0])
    yield from map(f, inp[1:])
```

```
# example C
def map(f, inp):
    if not inp:
        yield []
        return
    yield f(inp[0])
    yield from map(f, inp[1:])
```

```
# example D
def map(f, inp):
    if not inp:
        return
    yield from map(f, inp[1:])
    yield f(inp[0])
```

```
# example E
def map(f, inp):
    if not inp:
        return
    yield f(inp[0])
    yield from map(f, inp[1:])
```

**Question 3: Backtracking with Tent Packing – see readme**

What are the success base case(s)?

What are the failure base case(s)?

What are the recursive case(s)?

Write a description of a high-level algorithm you could use to solve the problem

**Question 4: More practice**

**# example a** Write the body of an infinite generator that will produce the desired output below

```
def fibonacci_generator(a=0, b=1):
```

```
    for i, fib in zip(range(9), fibonacci_generator()):
        print(f"fib({i})={fib}")
```

```
# fib(0)=0 fib(1)=1 fib(2)=1 fib(3)=2 fib(4)=3 fib(5)=5 fib(6)=8 #fib(7)=13 fib(8)=21
```

**# example b** -- turn this generator into a regular function

```
def flatten(lst):
```

```
    for item in lst:
```

```
        if isinstance(item, list):
```

```
            yield from flatten(item)
```

```
        else:
```

```
            yield item
```

```
nested_list = [1, [2, [3, 4], 5], 6, [7, 8]]
```

```
print(list(flatten(nested_list))) # [1, 2, 3, 4, 5, 6, 7, 8]
```

**Question 5: BFS vs DFS + recursion**

What will the program below output?

```
def dfs(graph, start):
    visited = set()

    def dfs_recursive(vertex):
        visited.add(vertex)
        yield vertex
        for neighbor in graph[vertex]:
            if neighbor not in visited:
                yield from dfs_recursive(neighbor)

    yield from dfs_recursive(start)

# Define a graph as an adjacency list
graph = {
    'A': ['B', 'C'],
    'B': ['D', 'E'],
    'C': ['F'],
    'D': [],
    'E': ['F'],
    'F': []
}

# Perform DFS traversal starting from 'A'
for vertex in dfs(graph, 'A'):
    print(vertex)

# For an extra challenge, write a recursive BFS version below. What will the #
printed output be now?
```

## Summary of Readings Since Exam 1

### Recursion

- base case, recursive case, combination
- recursive cases are smaller than original
- helper functions

### Recursion and Iteration

- some problems naturally iterative, some recursive
- recursive patterns
  - list-like: first/rest
  - tree-like: children
  - graph-like: neighbors
- recursive helper to accumulate partial results
- recursion for DFS
- generators

### Recursion with Backtracking

- a generalized graph-like search with constraints
- typical structure
  - success base case
  - failure base case
  - recursive case that reversibly tries possibilities

### Custom Types

- the power of abstraction
- Python's class keyword and the environment model
- class vs instance attributes / variables
- two scoping paths: variable lookup through frames, attribute lookup through dot notation / classes

### Inheritance

- subclasses and instances
- inheriting vs overlaying methods, leveraging polymorphism
- lifting shared behaviors to superclasses

### Functional Programming

- converting imperative-style loops to functional-style recursion
- converting classes to nested functions
- memoization